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Signals and Systems

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Final Exam

* 1. The given system is non-causal because it does not satisfy the condition for causality:  
     For the given system, we can see that h(-1) = -1 and that h(-2) = 1.
  2. Any system is said to be BIBO stable if  
     For the given system, we have:  
     Therefore, the given system is BIBO stable.
  3. A picture containing antenna, object

     Description automatically generatedTo find the output y(n), we can write:  
     Therefore:  
     The output graph of y(n) is as follows:

1. First, let’s plot the graph of x(n):  
   A picture containing object, antenna

   Description automatically generated  
   Next, let’s consider the equation  
   From this, we can say:  
   Thus, we can write our impulse response h(n) as:
2. Given   
   If we take Laplace transform of this equation, we get:  
   1. To determine output when , we know that:  
      Because f0 < W, it allows the original signal
   2. To determine output when , we know that:  
      Because f0 > W, the signal will not be allowed into the circuit
3. From the diagram, we can state the following:  
   Next, we can apply the z-transform:  
   We can rewrite equation 1) to say  
   Similarly, we can rewrite equation 2) to say  
   We can then say:  
   Next, we calculate our transfer function for k=2:  
   We can finally calculate the roots of our denominator for our transfer function at k=2:  
   We can see that the pole lies outside the unit circle of the z-plane, therefore the system given is not BIBO stable at k=2.  
     
     
   1. If the system is both causal and stable, then all the poles of H(z) must lie inside the unit circle of the z-plane because the ROC is of the form (For right-handed signal). Since the unit circle is included in the ROC, then we must have . Given:  
      Next, we can calculate for the ROC of our system:  
      Therefore, the given system is both causal and stable for -1 < a < 1.
   2. Given:If we take the inverse z-transform of the above equation, we will get:
   3. As given, the transfer function for a causal, first order, finite impulse response filter in the z domain is of the form  
      To analyze this in the frequency domain, we can say  
      At :  
      At :  
      Solving equations 1) and 2) above, we get  
      Substituting back in, we get  
      Therefore, we can say that a causal, first-order, finite impulse response filter has been designed with
   4. Taking the inverse z transform of the H(z) we solved above, we get  
      Additionally, we know that our input . We now find output y(n) when x(n) has the above values:  
      We can say applying the filter obtained in part a) to the given input x(n) gives us the following output
4. Given:  
   We can rewrite the above to say:  
   1. We now ensure requirement 1 of the question is met.  
      For the 2 samples below, we will use the following:At n=0:  
      At time of 0.02 seconds (because of y(n) 36% decay), we have our second sample:
   2. We now ensure requirement 2 of the question is met.  
      Substituting our above condition in, we get:  
      Therefore, our system response is:  
      or
5. Given:  
   We can replace with:  
   At   
   At   
   But we know that :  
   But we know that :  
   For given filter to be stable, our ROC must include unit circle. For given filter to be causal, our ROC should be outer of our outermost pole. The pole is at . Our ROC would therefore be . For our ROC to include unit circle, .  
     
   Let’s analyze at to see stability:  
     
   At :  
   Here, we have . This ROC is stable because it includes the unit circle.  
     
   At :  
   Here, our ROC is and does not include the unit circle, therefore it is not stable.  
     
   The only stable option for a1 is .  
   Thus, we have   
     
   We have calculated the coefficients such that our filter is stable and causal and fulfills the two criteria: